

An Overview of LeActiveMath — Language-Enhanced, User-Adaptive, Interactive eLearning for Mathematics

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The LeActiveMath system is an innovative third generation eLearning system for school and university level learning, for classroom teaching as well as for self-study. It was developed in close collaboration between experts in math education and computer science and offers new ways to learn mathematics based on modern pedagogical approaches by adapting itself to the goals of the individual learner, to her/his competencies and motivation. Interactive tools serve to deepen the mathematical knowledge.

The workshop at DES-TIME 2006 extended Prof. Dr. Reiss' keynote and offered the possibility to work with a prototype version of LEACTIVEMATH. Special emphasis was put on the adaptivity of the system, especially the dynamic course generation.

Keywords

pedagogical strategies, course generation, learner model, higher education, mathematics education, self-regulated learning, eLearning, interactivity, adaptivity

1 Introduction

LEACTIVEMATH (Language-Enhanced, User-Adaptive, Interactive eLearning for Mathematics) is an interdisciplinary project, which is funded under the 6th framework of the European commission. Its objective is to develop and evaluate an internationalized Web-based intelligent eLearning system for mathematics.

The LEACTIVEMATH system is distributed under a open-source license and is freely available for non-commercial purposes.

1.1 Overview of the LeActiveMath project

The LEACTIVEMATH system is based on an existing eLearning system (ActiveMath), which has been jointly developed by the German Research Center for Artificial Intelligence (DFKI) and the Saarland University (UdS). Starting in January 2004 and

with a duration of three and a half years, the LEACTIVEMATH project has the goal to develop and evaluate a truly interactive and intelligent Web-based eLearning system for mathematics. In addition to DFKI and UdS, the consortium includes the University of Edinburgh (United Kingdom), Technische Universiteit Eindhoven (Netherlands), Ludwig-Maximilians-Universität München (Germany), University of Glasgow (United Kingdom), and Universidad de Malaga (Spain). Additionally, two private companies are contributing to the LEACTIVEMATH project: the European Research and Project Office GmbH (Eurice) and Ernst Klett Verlag, the largest German educational publisher.

LEACTIVEMATH is an eLearning system for mathematics. Its content focuses on differential calculus and is hence intended for secondary (higher) education, resp., university level. The first two and a half years of the project were mainly devoted to the authoring, translation and improvement of the content and the development and implementation of the different novel components of the system, such as a tutorial component which assists the learner in creating her own books, especially designed for her specific needs; and a learner model which not only models the learner's knowledge with respect to each mathematical concept but also his motivational and affective state, as well as an "Open Learner Model", i.e., an interface which the learner can use to inspect his learner model.

In Section 2.2, we will describe these components in detail and provide an overview of other features of the LEACTIVEMATH system.

The remaining year of the project will serve the integration of all components, but mostly focus on several large scale evaluations of the system, in high school as well as in university context. These evaluations aim at testing the efficiency of this eLearning environment, in multiple aspects: not only with respect to increasing the learner's knowledge but also with respect to improving her motivation and affection. We will discuss the evaluations in Section 3.

1.2 A First Look on LeActiveMath

A demo version of the system is accessible online;¹ for a first (static) impression see Figure 1. The screenshot shows the structure of a "book" or "course" on the left. Like a table of contents of a real book, it consists of chapters and foldable sections and pages. The shown page here is part of the book "Highschool Content", Chapter 4, "Differential calculus", Section 4.2, "Introduction into derivatives". Its name is repeated on top of the main window, directly below the menu bar. The menu bar allows direct access to the main features of LEACTIVEMATH, such as the search. At the right-hand side, a navigation bar indicates that the main window displays page 85 of 166 pages in that book.

Each page is an arrangement of single items (separated by green rulers). To the left of the title (here: "The slope on a section of the Tour de France") a small colored triangular icon indicates the type of the item, in this case the turquoise "E", which symbolizes "example", cf. Section 2.1. The other icon (on the right) with the pencil opens a list of possible actions for this item, which e.g. allows the user to add a note or

¹<http://leam-calculus.activemath.org>

The screenshot shows the LeActiveMath interface. On the left is a navigation menu with categories like 'Highschool Content of LeAM_calculus', '1 Basics', '2 Sequences, series, and', '3 Functions', '4 Differential Calculus', and sub-sections like '4.1 Difference quotients', '4.2 Introduction into deriv', '4.3 Differentiation rules', '4.4 Derivatives of various', and '4.5 Theorems on different'. The main content area is titled 'From the average slope to the derivative (alternatively)' and contains a lesson titled 'The slope on a section of the Tour de France' with two stars. The lesson text describes a section of the Tour de France between kilometers 126.5 and 162, starting at a height of 588 m and ending at 1442 m, with a height difference of 854 m. A graph shows a mountain profile with a red line segment representing the section, a magnifying glass over it, and a right-angled triangle with a hypotenuse of 35.5 km. Below the graph, the text provides the length of the distance (35500 m), height of the starting point (588 m), height of the end point (1442 m), and height difference (854 m). It then calculates the distance in the plain (35490 m) and the average slope $m = \frac{854}{35490} = 0.024$. The footer includes 'Your Opinion' and 'Computations of slopes with bigger exactness'.

Figure 1: The LEACTIVEMATH system: a section on “the average slope”

dynamically include content in the book, cf. Figure 6. Finally, on the right of the title, the two yellow stars indicate that this examples is an “easy” example, on a scale ranging from one (very easy) to five stars (very difficult) .

In comparison to the previous system “ACTIVEMATH”, the graphical user interface (GUI) has been improved significantly, due to usability tests performed by the Fraunhofer Institute.

Not visible in the static screenshot of Figure 1 is the *adaptivity* of the LEACTIVEMATH system, which will be described in detail in Section 1.3. Adaptivity means that the content and overall system is adapted to the individual learner and his learning goals. Such a feature requires identification, hence learners have to register and state some personal properties — at least if they want to work on exercises or create personalized books, etc. Registration and log in guarantee that the system recognizes the user when she returns at a later time and remembers her previous interactions with the system.

1.3 Distinguished Features of LeActiveMath

Two main design goals guided the development of LEACTIVEMATH: *flexible usage* and *adaptivity*. Flexible usage means that LEACTIVEMATH is usable for high school and university students and supports both classroom and homework settings. Existing learning tools, such as CAS should be easily integrated. Adaptivity is achieved by the system selecting those learning materials most appropriate to the learner’s current goals. Let us briefly describe these two features (for more details, see the following sections).

Flexibility is achieved by a user interface that was especially designed to be usable by high school and university students. All features are documented in a help function and all user interface elements are explained by using tooltips. For each learner, the system automatically selects the content appropriate to her educational level.

Additional flexibility is achieved by the easy integration of learning tools. In the last years, a vast number of tools for learning have been developed: search tools, exercise back-engines, simulation tools, mind map tools, modeling tools, assessment tools, CASs, etc. The LEACTIVEMATH system makes the integration of such tools easily possible by providing an open architecture that, among others, relies on a semantic representation of mathematical formulas using OpenMath.² This semantic representation is also used by the search facilities.

Further flexibility covers internationalization. The complete content developed in LEACTIVEMATH is available in English, Spanish, and German, as well as the user interface of the system. New languages can be added without much effort besides the translation itself.

The second, related prominent feature of the LEACTIVEMATH system is adaptivity. It includes adaptation to modern pedagogical approaches as the competency-levels as they are used in the PISA studies and adaption to the learner's profile, goals and context. Adaptivity is realized by the tutorial component. This component supports the learners while they interact with the learning materials: they can specify their learning goals and a course generator automatically assembles a sequence of learning materials that fits best the learner's goal, capabilities, and preferences. Additionally, an exercise suggestion agent dynamically selects those exercises best suited for the learner. In case learning problems arise, a suggestion agent provides hints to the learner how to overcome the difficulties. All suggestions and hints are based on well-founded pedagogical expertise. By new forms of interactions, the learners, too, can access the pedagogical knowledge and actively extend the course. Section 2 provides more details.

Adaptivity requires that the system knows what the learner knows. For that purpose, a component called *learner model* stores beliefs about the capabilities of the student. The learner model's estimations of the learner's capabilities provide valuable information to the learner herself and hence is directly visible in the user interface.

Indication of knowledge Figure 1 shown previously contains the GUI from the perspective of a novice user. Each grey box in the table of content on the left represents a page of the book and indicates by its color the student's knowledge of the topics on that page — or more precisely, the estimation on the student's mastery held by the system. These values are primarily computed using the success rate while solving exercises. In case the learner has not solved any exercises yet, the system cannot calculate such an estimation, which is then indicated by the color grey. In contrast, Figure 2 shows the table of content of the same book for a more advanced user, with a visible difference in the colors. The figure also provides an overview of the color scheme of the boxes. This color scheme is inspired by the color scheme of traffic lights and changes from red to

²<http://www.openmath.org>

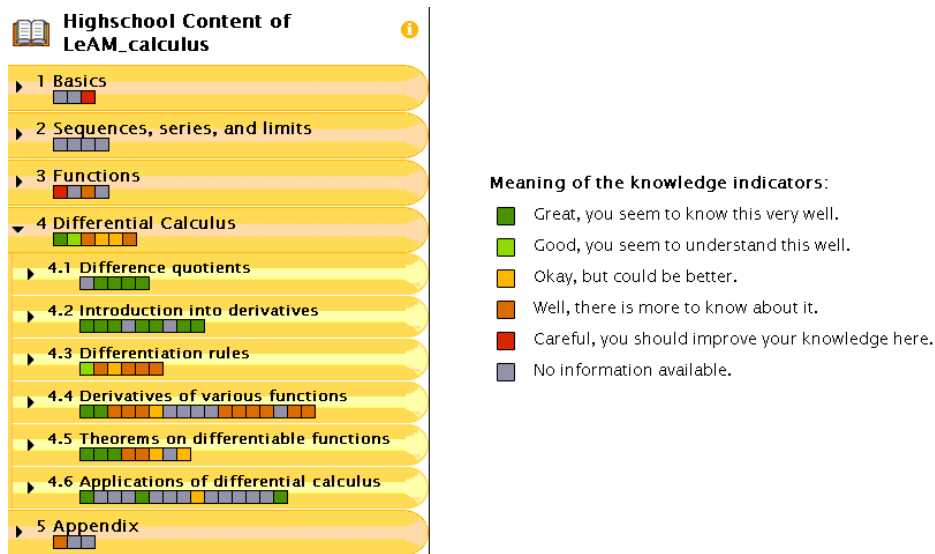


Figure 2: The LEACTIVE MATH knowledge indication bar

green with three intermediate steps.

The learner model is able to draw inferences based on the user’s interactions. Therefore, a non-grey color doesn’t necessarily imply that the user has already seen the corresponding page. For instance, if a learner immediately starts with the chapter on derivatives, (thus skips the chapter on difference quotients), but still is able to handle derivatives very well, then the learner model will assume that she also can deal with difference quotients, since the latter are the basis of derivatives. This means that mastery values are propagated using dependencies between the content, and, as a consequence, the chapter on difference quotients will change color.

Yet, all technological aspects alone would not guarantee the high flexibility of the system. To that end, one also needs to have a sufficiently large amount of content material. Moreover, this content has to be of an elaborated design embedded in a sensible pedagogical framework to allow for a wide range of usage in school and university.

1.4 Pedagogical aspects

LEACTIVE MATH’s innovative features serve a moderate constructivist and competency-based pedagogical approach.

The “PISA paradigm shift” in educational research aims at making the students gain competencies in mathematics that they can apply in everyday problems, instead of drilling pure calculation skills. The expression “life long learning” is frequently used in the current pedagogical discussions and “learning to learn” is considered to be of main importance. The belief behind this is that in order to assert one’s position in school and in the job one has to be able acquire knowledge self-dependently and flexibly.

“We live in a time of extraordinary and accelerating change. New knowledge,

tools, and ways of doing and communicating mathematics continue to emerge and evolve. The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase.”

What can be derived from this quotation by the National Council of Teachers of Mathematics (2000) is that mathematics is clearly characterized as a science relevant for daily life. This as a matter of course means also mathematics is subject to changes: Mathematics has to adapt to present and future challenges in social and working life. Certain mathematical subject areas will become less important others will gain attention. Probably also new ones will emerge. Such changes in the focal points of a subject consequently lead to the requirement of new, adequate learning material.

In *LEACTIVEMATH* we decided to follow a problem based approach that requires and strengthens argumentation skills, modeling abilities and other mathematical competencies (for an overview of the mathematical competencies cf. (Niss, 2002)). Other important abilities to be developed by the students are (heuristic) strategies and reflection capabilities. They are taken into consideration in *LEACTIVEMATH* as well

The following description compasses the main pedagogical aspects of *LEACTIVEMATH* and the way they are realized:

- active role of the learner and interactivity — Various tools support the learner in actively interacting with the system, e.g. the different exercise formats ranging from multiple choice questions to concept map exercises³ A number of hints have been authored for each exercises. Those hints have been classified in those that give away parts of the solution and those that do not. If the learner requests help for the first time within an exercise, he will only receive a general advice that helps him enter the problem solving process. This general advice may even be an easier task of the same type as the original one.
- comparable learning outcomes via educational standards and competencies — Educational goals of schools are often relatively general statements about the knowledge, abilities and skills that schools are expected to impart. They usually include expectations for the development of the learners and are connected with a certain view on the significance of a subject. The role of the educational standards and the competencies⁴ is explained in (für Bildung und Forschung, BMBF, p. 16):

Educational standards put goals into concrete terms in the form of competency requirements. They stipulate the competencies students must

³In those exercises the learner can usually pull “concept nodes” from a pallet into a working space. There he can draw edges between those concept nodes he considers to be related. What makes the tool stand out is that it is able to check whether the edges are correct or not.

⁴At this competencies are ”cognitive abilities and skills possessed by or able to be learned by individuals that enable them to solve particular problems, as well as the motivational, volitional and social readiness and capacity to utilize the solutions successfully and responsibly in variable situations” Weinert (2001).

possess if key learning objectives are to be considered achieved. These requirements are organized into competency models, which describe aspects, levels and the developmental trajectories of competencies.

All examples and exercises written for LEACTIVEMATH were annotated with competencies and competency levels.

- offering learning opportunities for self-guided learning — The pedagogical strategies serve this important purpose. They are designed according to certain scenarios which correspond to high-level learning goals. The student first selects the scenario, then decides which topics to focus on. After such “planning” activities she receives personalized learning material including dynamically produced narrative bridges such as introductions or summaries, as well as personalized suggestions. This feature is designed to help in classroom differentiation as well as in self-learning. The different scenarios will be described in Section 2.2

2 Detailed Description of LeActiveMath

The LEACTIVEMATH system can be visualized as a triangle consisting of *content* annotated with metadata, *adaptivity*, mainly realized by the tutorial component, and *learner modeling* offering detailed information about the users. In this section, we will describe each of these parts in detail and explain how they interact.

2.1 LeActiveMath’s Mathematical Content

The content focuses on differential calculus in one variable. Due to the limited resources and project duration, the consortium agreed to restrict authoring on that particular area. Still, a large amount of learning material was produced. The content of LEACTIVEMATH compasses texts, formulas, pictures resp., photos (approx. 600) and interactive elements (java applets, approx. 70). To allow for adaptivity, the whole content is consequently broken down into single items such as symbols (resp., concepts) and their definitions, assertions (theorems, corollaries, lemmas), proofs, examples, exercises, and finally connecting texts such as introductions, motivations, and notes. Although the partners (mostly Munich) authored more than 400 exercises and examples, this is still not sufficient to cover each possible need for an exercise or example with a specific topic, difficulty, competency and competence level. In this respect it might be of future importance that LEACTIVEMATH is an open system with an exercise repository waiting for contributions of authors yet to come.

Each of the items has at least English, German, and Spanish titles and texts. Some items are additionally available in French, Italian, and Chinese. Transformed into a static text book, the whole content authored so far exceeds 500 pages – excluding the interactive exercises with all their hints and feedback adapted to the user’s input. Note that this is indeed another feature of LEACTIVEMATH: every book, whether pre-recorded or generated on user request, can be presented in three formats: in HTML or MathML

(which produces good looking math formulas) to be viewed in a browser or as a pdf file that can be printed with high quality layout. The latter possibility eliminates a typical disadvantage of a learning software as it can be used *without* a computer.

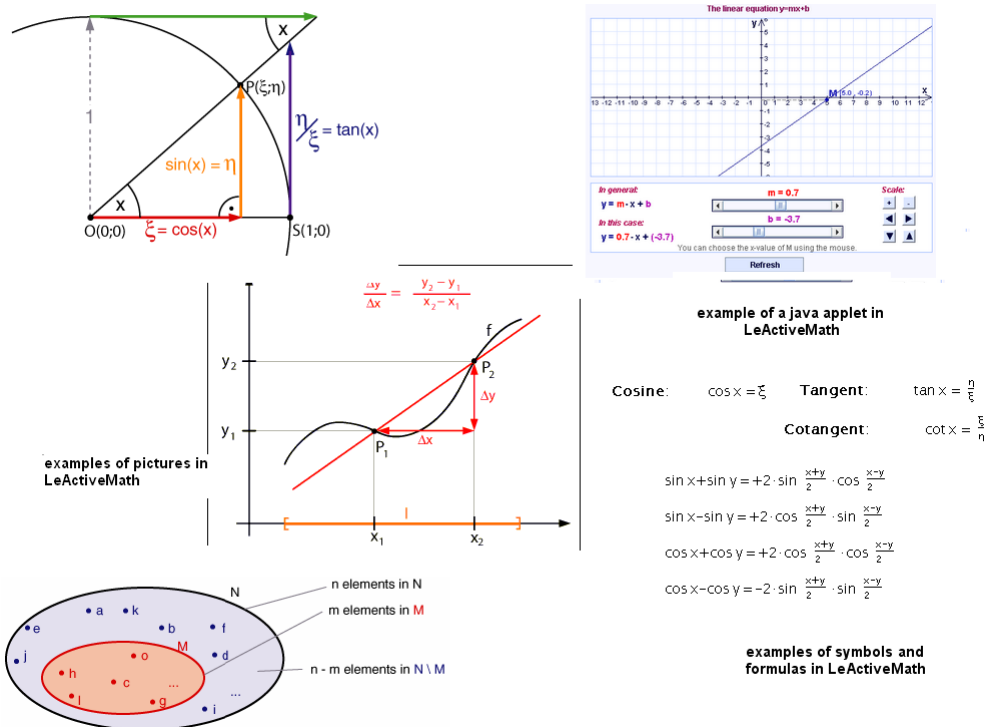


Figure 3: Examples of LEACTIVE MATH's content

As was mentioned before, each content item also carries a set of metadata used to characterize it. These metadata not only specify the author(s), translator(s) and other contributors but also provide semantical and pedagogical informations.

The set of metadata for exercises differs from the ones for theorems and contains e.g., a value for the estimated time needed to interact with the item or for its abstractness. The metadata are used by the Tutorial Component for the book creation. Perhaps most important are the specified relations between items, like which concepts are trained by some exercise, which concepts a theorem involves (and hence depends on), or which theorem or definition an example is illustrating. Similar dependencies also exist between different concepts: the “higher derivative” depends on the “derivative function”, which in turn depends on the “derivative of a function at a given point”. The latter depends on the concepts “function”, “difference quotient”, and “limit”, in order to mention only some of the relations collected in the metadata.

Pedagogical informations specified in the metadata are, e.g.,

- intended learning context (e.g., “higher education” or “first year university”),

- field of interest (e.g., physics, computer science), which is especially relevant for examples and exercises that involve developing a mathematical model.
- representation (“verbal”, “visual”, “symbolic”, and/or “numeric”),
- abstractness (“concrete”, “neutral”, or “abstract”), and
- typical learning time (important, e.g., for exam simulations).

Moreover, exercises and examples carry additional metadata on

- difficulty (graduated in five steps from “very easy” to “very difficult”),
- competency required, resp., trained, and
- competence level (in four levels ranging from “elementary” to “complex”).

Following the competency model in Niss (2002) we singled out the following eight competencies:

- mathematical thinking
- argumentation and proof
- modeling
- solving
- use of mathematical representations
- use of symbolic and formal language
- mathematical communication
- use of mathematical tools

The specification of all these competencies for each exercise and examples enables the Tutorial Component to create individualized workbooks on user request, where she can individually and specifically train her own mathematical competencies.

Finally, all texts are enriched by approximately 30.000 references: e.g., clicking on the link underlying the phrase “sum rule” will take the user automatically to the search window displaying the theorem that contains the corresponding differentiation rule. In this way these references support the user in rehearsing concepts she is still not sufficient familiar with. A study of the data collected in the midterm evaluation (Section 3.1) showed that approximately 90% of all calls for the search function could be attributed to these textual references.

The text describing the content was written with keeping the target group in mind. For school students, this language has to be familiar, and in particular, not too formal but with the necessary precision.

With respect to our aims (making processes transparent, prompting metacognitive skills) we not only present complete solutions but also explain individual steps of solutions, resp. what the next steps are about. We also included prompts on what one knows so far, resp., what is still missing/to be done next.

Referring to feedback and hint texts we always tried to vary in formulations and not to write too long texts. As shown in several studies (for a meta-review see Jakobs (2002)) nobody is going to read feedback till the end, if texts are too long. Some authors think “too long” begins with a 4th line.

Especially in MCQ-, but also in FiB-exercises we tried to address the typical misconceptions. This should guarantee that the user is not left with the usual default feedback but receives an adapted feedback depending on his actual errors, which is an important point to increase the user’s motivation.

2.2 Adaptivity in the LeActiveMath System

The Tutorial Component realizes the adaptivity, reactivity, and integration of tools: it selects and assembles learning material adaptively relative to the learner’s goal, competencies and preferences; reacts by monitoring the learner’s interactions with the LEACTIVE MATH system and providing supportive feedback and it responds to requests of the learner; it integrates learning supporting tools developed in LEACTIVE MATH by seamlessly blending learning materials and the tools in courses presented to the learner.

The Tutorial Component consists of three main sub-components, which each serves a specific functionality: a Course Generator adaptively selects learning material that supports learners in achieving pedagogical tasks; a Suggestion Agent monitors a learner’s behavior and provides remediating feedback in case potential problems are detected; an Exercise Sequencer adaptively selects exercises leading the learner towards a higher competency level.

Pedagogical Strategies The Tutorial Component uses formalized pedagogical knowledge for its decision making. Pedagogical strategies are teaching strategies that integrate viewpoints from pedagogy, educational psychology, and mathematics education. Central to a strategy is the learning theory it is based on. The learning theory influences the “look-and-feel” of the learning materials as well as the interactions between the eLearning system and the learner.

LEACTIVE MATH is based on *moderate constructivism*. Relying on modern brain and mind research, this theory claims that learning processes can only be initialized by learners themselves, by their active (constructivist) operation on the subject domain. Hence, learners play an active role in their learning processes and are responsible for the outcome of their learning processes to a major extent.

As a consequence, learners need to learn more than subject matter, and need to be supported in structuring their learning activities and developing strategic competence. Therefore, teaching should serve the overall goal that students become autonomous and self-regulated learners.

We identified six major learning goals and used them to define *scenarios* in which the strategies are applied. Scenarios describe typical learning phases, like *introduction* or *practice* (cf. Zech, 2002, p.181ff). Those phases appear in lessons to differing extent, depending on context and learning goals and, of course, on the learner's needs. Therefore, there is one most adequate (main) strategy for a learner with a specific learning goal. It determines the basic structure of a course by, for instance, prescribing that the presentation of an example should follow the presentation of a definition. The strategies are adaptive, that is, different materials can be selected by taking information from the learner model into account, for instance, the learner's motivational state or his success in dealing with previous exercises. The scenarios are the following:

Discover Books created with this scenario will contain content that helps the learner to understand the selected topics in depth. It contains the prerequisites she needs to understand and it is adapted to her knowledge state.

Rehearse This kind of book addresses weak points of the learner and trains her knowledge in these areas.

Connect In this book the connections between the selected topics are illustrated, adapted to the learner's knowledge.

Train competencies This kind of book contains exercises that train specific competencies.

Workbook This kind of book contains exercises that aim at increasing the learner's current knowledge state.

Simulate an exam This kind of book contains exercises that can be solved within a timeframe set by the learner. Like a real exam, the exercises will have different difficulty levels.

To summarize, the scenarios determine which content is selected and presented to the learner. Let's now take a look at the separate components of the Tutorial Component.

Course Generator The *Course Generator* dynamically assembles a sequence of learning materials according to the above scenarios. During the assembly, it uses information about the learner and the available content and hence queries the learner model, the LEACTIVE MATH content repositories and potentially other available third-party repositories. Once a course is assembled, it is presented to the learner.

Exercise Sequencer The *Exercise Sequencer* provides the learner with a sequence of dynamically selected exercises that leads him towards a learning goal. It selects an exercise, presents it to the learner in a GUI separate from the current course, and depending on the learner problem-solving success provides feedback and terminates or selects a new exercise, thus starting the cycle again. The selection algorithm is based on competency levels, and has the goal to lead the learner from his current competency level to the next one.

Suggestion Agent The *Suggestion Agent* monitors the user’s interactions and offers feedback in case problems are diagnosed. Suggestions consists of navigational hints or learning materials. For the latter case, the Suggestion Agent uses course generation

Learner Interface In addition to the generation of course that consists of adaptively selected learning materials and learning supporting tools, the Tutorial Component offers two basic features that support the learner’s active engagement in accessing the content: user-triggered course generation and course extension. In LEACTIVEMATH, course generation is started by the learner. A wizard guides the learner first through the selection of the scenario and then through the selection of the content goals. The assumption is that this way the learner becomes aware of his learning goals and trains to articulate them. Moreover, if the learner wishes to see additional learning material about a concept in course, she can use an *item-menu* to select one of several actions by selecting them from a drop-down list. Then, the request is processed by the Tutorial Component, the resulting learning objects are presented to the learner and integrated in a course upon her request.

Figures 4 and 5 illustrate the course generation process; Figure 6 contains an example of the item menu.

Additional Components of LeActiveMath Besides the tutorial component and the learner model, the LEACTIVEMATH project aimed at integrating or developing several other tools which support the learning process. These include

- an assembling tool that allows the learner to “bookmark” those items that were most helpful or interesting during the lesson interaction;
- a Natural Language Dialogue Manager that allows the learner to type in questions in regular language as well as mathematical formulas, and have the computer generate language-based responses and explanations based on reasoning automatically about the correctness or incorrectness of the learner’s statements;
- a Concept Mapping Tool that allows the learner to visualize relationships between concepts that are not immediately obvious through the lessons themselves;
- the Siette Assessment Tool that dynamically creates tests to assess the learner’s performance. It adaptively poses problems to the learner, allowing for a more accurate assessment of their capability for problem solving.
- a sophisticated semantic search function;
- a function plotter;
- a Computer Algebra System (CAS) that can be accessed by learners via an input editor for formulas and by exercises and other components via OpenMath expressions.

Please **select the type of book** you want ActiveMath to create for you. Whether you want to prepare for an exam or discover a topic in depth – choose the type of book from the list that best suits your needs.

Type of book:

- Discover:** *This kind of book will contain content that **helps you to understand** the selected topics in depth. It will contain the prerequisites you still need to understand and it will be adapted to your knowledge state.*
- Rehearse:** *This kind of book **tackles your weak points** and will train your knowledge in these areas.*
- Connect:** *This kind of book **illustrates the connections** between the selected topics, adapted to your knowledge.*
- Workbook:** *This kind of book **contains exercises** that aim at increasing your current knowledge state.*
- Train competencies:** *This kind of book contains exercises that **train specific competencies**.*
 - Think mathematically: Solve problems mathematically: Use mathematical representations: Deal with symbolic and formal elements of Mathematics: Model mathematically: Argue mathematically: Know how to use tools and aids: Communicate on Mathematics:
- Simulate an exam:** *This kind of book contains **exercises that can be solved within a timeframe** selected by you. Like a real exam, the exercises will have different difficulty levels.*
 - Simulate an exam, 30 minutes; Simulate an exam, 45 minutes; Simulate an exam, 60 minutes;
 - Simulate an exam, 90 minutes.

Figure 4: The Course Generation

Step 3 of 4: Topics

In this step, you select the **topics** your book should be about. Navigate through the content by clicking on the headings until you the topics you want to learn about. Then, select one of them.
 Dont worry if you have never heard about a topic. ActiveMath will automatically include items that you need for your understanding, if necessary.
 Please be aware that there may be not enough content available for all combinations of scenario and topics, even though most of the time it works fine. One topic that always works fine is "the derivative function".

■ Contents of differential calculus for the composition of individual courses

1 ■ Introduction into differential calculus
 This chapter leads from the difference quotient to the derivative. It gives the most important interpretations on this (as slope, resp. rate of change).
 1.1 ■ Average slopes, rates of change, and further difference quotients 1.2 ■ (Actual) rates of change and slopes, derivatives 1.3 ■ Tangent and perpendicular ...

2 ■ Differentiation rules
 It's very troublesome to compute derivatives from first principles. Instead, it's much easier to use ready made formulae for this. These differentiation rules are the topic of this chapter.
 2.1 ■ Differentiation according to the sum rule 2.2 ■ Derivation of power functions with natural numbers as exponents
 2.3 ■ Differentiation according to the product rule ...

Figure 5: Selecting the topic(s) of a personal book

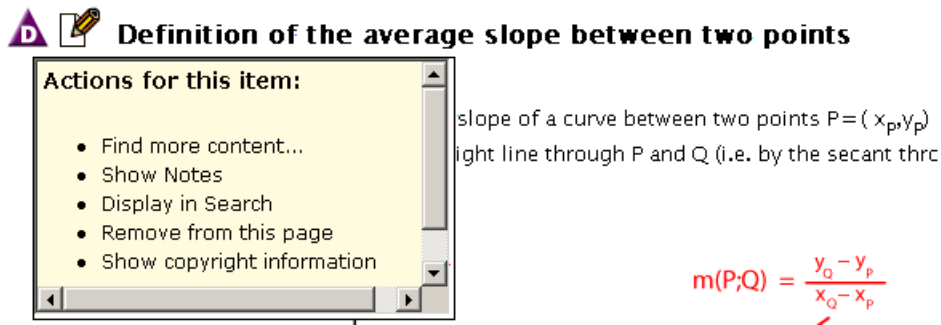


Figure 6: The item menu attached to learning elements

2.3 Learner Modeling

The learner model stores beliefs about the capabilities of the student. This includes modelling beliefs about student's domain knowledge as well as the student's meta-cognitive state and beliefs about the student's situational awareness, emotional and motivational states, and attitudes. Additionally, the student's interactions with the system are stored in a learner history.

An *Open Learner Model* provides ways of viewing the model, as well as mechanisms that allow the student to change the model's contents by a negotiation process. It supports the learner in continually managing and reflecting her process of understanding new concepts and of relating them to other concepts, to theorems, examples or properties.

3 Evaluation

The overall aim of the application of LEACTIVEMATH is to support students in structuring learning processes, and to offer learning ways that are as individual and appropriate to each learner as possible. The intention of the midterm evaluation is to investigate the extent to which the current version of LEACTIVEMATH indeed achieves this aim.

3.1 Design and Purpose of the Midterm Evaluation

Though not all components could be included in all testings, we designed a frame for this evaluation phase where we managed to cover our main questions across the different sub-evaluations. The most relevant questions at this time were

- the general learning effectiveness of the system
- the acceptance of the system
- the appropriateness of the metadata and
- the detection of errors.

The first and the third aspect are harder to evaluate than the other ones and require further explanations: as will be described later in this section we made 11th graders work with LEACTIVE MATH. The students were at the beginning of the study completely unfamiliar with differential calculus. Therefore we postulated that the students would have learned some basic concepts in this field of calculus after the first lessons. A test was designed with items around those central concepts covering the most important procedures and ideas. The items had all been taken from the existing LEACTIVE MATH content and were grouped according to competencies and competency levels required. This way, we tried to measure the learning outcome and derive the learning effectiveness of the system.

The term *metadata* stands for all data that annotates the learning objects as described in Section 2.1. We are able to verify the metadata's appropriateness using the students' learning outcomes. Partially the stored data of the learner, the so called learner history gave important indications for adaptations to be done, e.g. in case of the estimated learning time, i.e. the time needed to perform a certain task.

Two evaluations, one of which we performed in five school classes in Bavaria and another in two seminars at the universities in Munich and in Saarbrücken in parallel, were the most important ones. In both groups we made the subjects fill a questionnaire with mainly closed answer formats and the opportunity to provide open feedback and suggestions in open questions. This feedback is believed to be most important for initiating improvements and increasing the extent to which LEACTIVE MATH fulfills the requirements set.

Tests in school The class evaluations were conducted by Munich in fall 2005. The 107 participants were 11th graders and beginners in a course on differential calculus. They were expected to answer a questionnaire on motivation and interest (duration approx. 10 minutes) at the beginning and at the end of a period of about 5 software-supported math lessons. Additionally, the students had to complete an achievement test (duration approx. 45 minutes). Both sources were paper & pencil test.

The first lesson introduced each of the participating classes in the LEACTIVE MATH system. It consisted in a presentation of the main features of the software, the students' registration in the system and the first steps they did on their own within the LEACTIVE MATH system.

During the sessions, students could optionally complete online questionnaires. Those online sheets were integrated into the system by three different sets of questions: One specifically inquired exercises, the second inquired examples, and the last captured a summarized assessment of the different components of the LEACTIVE MATH system. Those question sets could be reached easily via button, either from the exercise or example on or after the log out on the subsequent page. Both, paper & pencil and online questionnaires complemented each other.

Test sessions in universities A course of lectureship students in Munich and a seminar of computer science students in Saarbrücken did a approx. 60 minutes session on

LEACTIVEMATH and answered an online questionnaire afterwards. The task for the 11 students to solve during the session was a refresh task on topics of differential calculus.

In the questionnaire they were not only asked about their own experiences and impressions towards the system and specific components but about their estimations about the system's value for first-time learners and about their suggestions.

3.2 Results

For the school classes did not work with a generated course but a predefined book and not all of them did the achievement test we will only report the results of the questionnaires in this article. The questionnaires have just been optional ones which led to the varying sizes of the sample.

Additionally, we will shortly summarize the feedback of the DES-Time workshop participants.

General Aspects School students appreciated to use software like LEACTIVEMATH in math lessons as it added some variety to the course. Other comments estimated LEACTIVEMATH valuable for being introduced into a new topic.

The university students, too, considered the software to be useful for learning differential calculus. However, some narrowed this opinion down to high achieving or highly motivated learners. One had hoped for an extended problem-based approach and criticized the very few examples of application of mathematics in other fields. Another one missed a formulary.

Hints and Feedback School students paid special attention on the “indication of knowledge” bar. By the time of the evaluation it covered only a scale of three colors (red, yellow, green). The students' complaints were mostly about the time it took to get ‘green’. This appeared to be too long for them. This point has not been mentioned at all by the university students.

Both, school and university learners, answered that they applied the hint-system frequently. Also in both groups a need for more concrete help was expressed⁵.

Navigation Almost only the university students provided comments about this feature. The school students just complained about the input editor resp. difficulties with typing in formulas in general. The university students, especially the ones from Saarbrücken criticized the structure and asked for shorter pages or a kind of restricted overview in order to keep the students motivated.

⁵At the time the evaluations were conducted only one hint per exercise was available to the students. Now, hints are scaffolded and appear in the order from a general hint on the first request to more detailed hints in later requests of hints.

Examples and Exercises Especially school students did not like the open exercises⁶ Exercises of other formats turned out to be a bit too difficult in some cases. The judgements on a five times Likert scale between 0 for “very bad” and 1 for “very good” were means of 0.69 for Multiple Choice exercises ($N = 31$, $SD = 0.30$) and $mean = 0.59$ ($N = 32$, $SD = 0.27$) for Fill-in-blank exercises.

The examples were found helpful by the students, and they saw an explanatory value in them. For some of them the amount of examples was not held to be sufficient. This opinion was shared by the university students who explicitly asked for a number example for each rule that is introduced. Though there was also a request for more similar exercises, we consider this to be covered by CAS supported exercises that work with (pseudo-)randomized numbers any time they are started. As LEACTIVEMATH was meant to support students acquiring flexible knowledge, the bigger part of the exercises is not intended to train standard procedures and therefore does not consist of problems which only vary at a surface level.

Pictures and applets The overall assessment especially for the pictures was positive ($mean = 0.73$, $N = 33$, $SD = 0.26$ and for applets $mean = 0.53$, $N = 29$, $SD = 0.19$), though there were single cases where numbers or signs within a picture turned out to be a bit difficult to read. This was mentioned by the school students. Furthermore especially from the students at the universities there came a request for more visualizations and explanatory material as examples in general.

3.3 Conclusions

Following the results of the midterm evaluation, a need for improvement on different components as well as for further content, especially explanations and less difficult exercises became obvious. Additionally it was necessary to restructure the content partially.

Through all groups very few learners commented on navigation issues or the quality of the search function. This could possibly indicate that some features (help manual, search function, etc.) were not used very often. So, we have to find ways to foster the usage of tools that we consider to be reasonable in learning processes. Special learning tasks, which require the usage of such features, could be a way to make the possibilities and advantages of those features more obvious.

4 Discussion and Further Work

Special effort is and will be put into the development and improvement of the pedagogical strategies. By the time of the first testing only the strategy “discover” was implemented and developed far enough. Improved versions of this strategy and versions of other strategies will be evaluated at universities through 2006.

⁶Those exercises can be discussed in groups but will not be evaluated by the system. In most cases they require multiple step solutions and modeling activities. Thus one rarely ends up with a clear and unique result.

4.1 Reactions of the Workshop Participants

Though the workshop only counted seven participants, lively discussions arose. In an online-questionnaire the workshop participants could answer some questions on specific components of LEACTIVE MATH, its user-friendliness and the generation of personal books. The questionnaire consisted of assertions to be judged using a five times Likert scale. The judgment turned out quite positive in general.

For there were only five people completed the questionnaire, a statistical analysis of the answers would not make much sense. Therefore, we describe the facets we emphasized before.

Components The participants had to judge the different components by ticking one box on a scale of five intervals ranging from “very bad” (encoded as 0) to “very good” (encoded as 1). In case the subject did not try that specific feature, it could check a corresponding box.

The “inspectable learner model” got a mean of 0.9, the “assembly tool” as well as the “hints and assistance” one of 0.75.

The schools students questionnaire also contained questions on different components. They judged the hint and feedback with a 0.31 which may - at least partially - be due to the non-working scaffolded hints, like mentioned before ??.

Application and user-friendliness In that part, the users were asked how well each statement applied, e.g. “The system is easy to learn”. The set of assertions was therefore given the fivefold Likert scale ranging from “not correct at all” to “absolutely correct” (0 is again indicating total disagreement; 1 means total agreement).

The participants were quite convinced about the effectiveness of the system as a whole (mean 0.88). They also found it easy to learn the system (mean 0.75) and easy to use it (mean 0.8).

Generation of personal books Examples for this set of assertions are “Being able to generate personal books is helpful” and “The book generation resulted in good books”. The fivefold scale ranged from “not at all” to “exactly” (again plus the additional option “not tried”).

This feature was considered to be helpful (mean 0.9). The item “I could understand the structure of the generated books (their sections and subsections)” got a mean that was even better (0.93). However, this rating has to be put relation to the limited time (two hours) the subjects worked with the system, which makes it very possible that the subject did not try all scenario. This is supported by the fact that for the following questions, which are relating to specific scenarios, at least one person decided to tick the “not tried” box. Nevertheless at least for some scenarios the participants gave very good marks.

General comments General comments that could be made in open questions met the point of contributing. A need for further content — on prerequisites and other mathematical areas, besides calculus was expressed.

4.2 Further Work

The remaining work to be done in the project LEACTIVEMATH on the one hand concentrates on the final evaluation and on the other hand on the needs that became obvious in the evaluation. As illustrated in Section 3 already a certain degree of guidance was requested by the learners and considered to be important.

Furthermore it might be desirable to have additional pedagogical strategies developed, e.g. a truly problem-based course or others that include resp. support more cooperative forms of learning.

The LEACTIVEMATH system is distributed under a open-source license and is freely available for non-commercial purposes. If you are interested in using the system or authoring content, please contact the authors or visit the Web site at <http://www.leactivemath.org>.

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